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An Optimal Control Law for
the One-Dimensional Case

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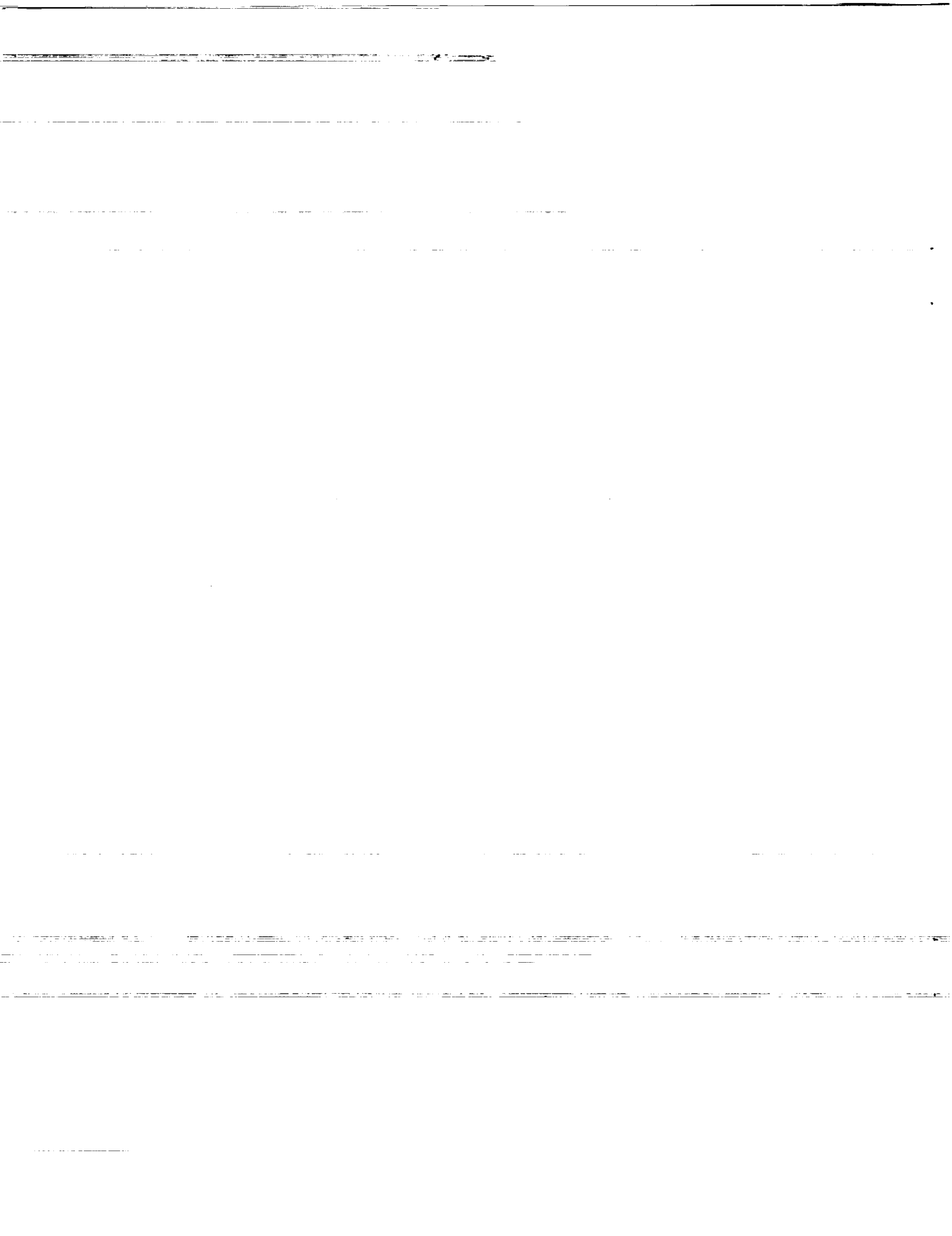*University of Virginia*
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MICROGRAVITY VIBRATION ISOLATION: AN OPTIMAL CONTROL LAW FOR THE

ONE-DIMENSIONAL CASE

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SUMMARY

Certain experiments contemplated for space platforms must be isolated from the accelerations of the platform. In this paper an optimal active control is developed for microgravity vibration isolation, using constant state feedback gains (identical to those obtained from the Linear Quadratic Regulator [LQR] approach) along with constant feedforward (preview) gains.

The quadratic cost function for this control algorithm effectively weights external accelerations of the platform disturbances by a factor proportional to $(1/\omega)^4$. Low frequency accelerations (less than 50 Hz) are attenuated by greater than two orders of magnitude. The control relies on the absolute position and velocity feedback of the experiment and the absolute position and velocity feedforward of the platform, and generally derives the stability robustness characteristics guaranteed by the LQR approach to optimality.

The method as derived is extendable to the case in which only the relative positions and velocities and the absolute accelerations of the experiment and space platform are available.

1. INTRODUCTION

A space platform experiences local, low frequency accelerations (0.01 to 30 Hz) due to equipment motions and vibrations, and to crew activity (ref. 1). Certain experiments such as the growth of isotropic crystals, require an environment in which the accelerations are less than a few micro-g's (ref. 2). Such an environment is not presently available on manned space platforms.

Since the experiment and space platform centers of gravity generally do not coincide, a means is needed to prevent a free-floating experiment from drifting into its own orbital motion and into the space platform wall. Additionally, most experiments will require umbilicals of some

sort to provide power, experiment control, coolant flow, communications linkage, and or other services. Unfortunately, such measures also mean that unwanted platform accelerations will be transmitted to the experiments. This necessitates experiment isolation. Passive isolators, however, cannot compensate for umbilical stiffness, nor can they achieve low enough corner frequencies even if umbilicals are absent. Therefore, it is essential to compensate this environment with active isolation means.

The problem then becomes to design an active isolation system to minimize these undesired acceleration transmission, while achieving adequate stability margins and system roubustness for the controller design. In addition, spatial and control energy limitations must also be accommodated. This paper will explore the optimal control problem of a microgravity experiment isolation from the low frequency range of disturbances experienced on the Shuttle and in the future Space Station Freedom Microgravity Modules.

2. MATHEMATICAL MODEL

The general problem has three translational and three rotational rigid body degrees of freedom. For simplicity, however, this analysis will consider only the one-dimensional problem. The general problem could be treated in an analogous manner. Let the experiment be modeled as a mass m, with position $x(t)$. Assume that the space platform has position $d(t)$, and that umbilicals with stiffness k and damping c connect the experiment and space platform. Suppose further that a magnetic actuator applies a control force proportional to the applied current $i(t)$, with proportionality constant α. Such a model is shown in figure 1.

The system equation of motion is

$$m\ddot{x} + c(\dot{x} - \dot{d}) + k(x - d) + \alpha i \quad 0 \tag{1}$$

Division by m and rearrangement yields

$$\ddot{x} = -\frac{k}{m}(x - d) - \frac{c}{m}(\dot{x} - \dot{d}) - \frac{\alpha}{m}i \tag{2}$$

In state space notation this becomes

$$\dot{\underline{x}} = A\underline{x} - \underline{b}u + \underline{f} \tag{3}$$

where

$$\underline{x} = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} x \\ \dot{x} \end{Bmatrix}, \quad \dot{\underline{x}} = \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} = \begin{Bmatrix} \dot{x} \\ \ddot{x} \end{Bmatrix},$$

$$A = \begin{bmatrix} 0 & 1 \\ -\dfrac{k}{m} & -\dfrac{c}{m} \end{bmatrix}, \quad \underline{b} = \begin{Bmatrix} 0 \\ -\dfrac{\alpha}{m} \end{Bmatrix},$$

$$u = i, \quad \underline{f} = \begin{Bmatrix} 0 \\ \dfrac{k}{m}d + \dfrac{c}{m}\dot{d} \end{Bmatrix}$$

The objective is to minimize the acceleration $\ddot{x}(t)$.

3. OPTIMAL CONTROL PROBLEM

The optimal control problem is that of determining the control current $u(t) = i$ which minimizes a suitable performance index

$$J = J(\underline{x},u,t) \tag{4}$$

for the system described by equation (3) subject to the state variable conditions

$$\underline{x}(0) = x_0 \tag{5a}$$

$$\lim_{t \to \infty} \underline{x}(t) = \underline{0} \tag{5b}$$

Another reasonable assumption is that $\underline{f}(t)$ is bounded, and it will be found mathematically advantageous (and only minimally restrictive) to assume that $\underline{f}(t)$ is also a dwindling function:

$$\lim_{t \to \infty} \underline{f}(t) = \underline{0} \tag{5c}$$

A quadratic performance index

$$J = \frac{1}{2} \int_0^\infty [\underline{x}^T W_1 \underline{x} + w_3 u^2]dt \tag{6}$$

has been chosen, as one that lends itself well to the variational approach to optimal controls, since an analytical solution is desired. The upper limit of the definite integral has been selected

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so as to yield a time-invariant controller. Here W_1 is a square 2 by 2 constant weighing matrix while matrix w_3 is a weighing constant.

Although, W_1 could be a full 2 by 2 matrix, for this problem a diagonal form has been employed for the sake of simplicity.

$$W_1 = \begin{bmatrix} w_{1a} & 0 \\ 0 & w_{1b} \end{bmatrix} \qquad (7)$$

The performance index consequently reduces to

$$J = \frac{1}{2} \int_0^\infty [w_{1a}x_1^2 + w_{1b}x_2^2 + w_3u^2]dt, \qquad (8)$$

so that each state is weighted independently.

If sinusoidal motion of the experiment is considered, so that

$$x(t) = B\sin \omega t$$

and $\ddot{x}(t) = \omega^2 x(t)$, the cost function can be expressed in terms of the acceleration and control as

$$J = \frac{1}{2}\int_0^\infty \left[\left(\frac{w_{1a}}{\omega^4} + \frac{w_{1b}}{\omega^2} \right) B^2 \ddot{x}^2 + w_3u^2 \right]dt \qquad (9)$$

It is apparent that this performance index convieniently weights accelerations at low frequencies much more than at higher frequencies.

4. SOLUTION

Finding the optimal control to minimize equation (4) is a variational problem of Lagrange, for which the initial steps of the solution are well-known (e.g., Elbert (ref. 3)). The variational approach is outlined below, following which the complications added by the non-homogeneous term $\underline{f}(t)$ will be addressed. Current optimal controls texts either assume that $\underline{f}(t)$ will be addressed. Current optimal controls texts either assume that $\underline{f}(t) \equiv 0$ (e.g., (ref. 3), p. 262) or require that it have a restricted range space (e.g., (ref. 6), p. 238). The solution that follows provides an analytical optimal control without imposing such restrictions.

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The argument of the cost function J from equation (4) is augmented by the Lagrange multiplier $\underline{\lambda}$ times the system equation of motion equation (3) where

$$\underline{\lambda} = \left\{ \begin{array}{c} \lambda_1 \\ \lambda_2 \end{array} \right\} \tag{10}$$

The result $\hat{J}$ can be expressed as

$$\hat{J} = \int_0^\infty H \; dt \tag{11}$$

where the Hamiltonian H is

$$H = \frac{1}{2} \left(\underline{x}^T W_1 \underline{x} + w_3 u^2 \right) + \underline{\lambda}^T \left(\underline{\dot{x}} - A\underline{x} - \underline{b}u - \underline{f} \right) \tag{12}$$

It is desired to obtain an optimal solution $u = u^*$ which minimizes $\hat{J}$.

The first variation of $\hat{J}(\underline{x}, u, \underline{\dot{x}})$ is

$$\delta \hat{J} = \int_0^\infty \left[\frac{\partial H}{\partial \underline{x}} \delta \underline{x} + \frac{\partial H}{\partial u} \delta u + \frac{\partial H}{\partial \underline{\dot{x}}} \delta \underline{\dot{x}} \right] dt$$

which is set equal to zero to minimize $\hat{J}$. However, integrating by parts,

$$\int_0^\infty \left(\frac{\partial H}{\partial \underline{\dot{x}}} \delta \underline{\dot{x}} \right) dt = -\int_0^\infty \underline{\dot{\lambda}}^T \delta \underline{x} dt$$

so that the above expression for $\delta \hat{J}$ becomes

$$\delta \hat{J} = \int_0^\infty \left[\left(\frac{\partial H}{\partial \underline{x}} - \underline{\dot{\lambda}}^T \right) \delta \underline{x} + \frac{\partial H}{\partial u} \delta u \right] dt = 0 \tag{13}$$

Both $\delta \underline{x}$ and δu are arbitrary variations, so $\delta \hat{J} = 0$ only if

$$\frac{\partial H}{\partial \underline{x}} = \underline{\dot{\lambda}}^T \qquad (14a)$$

$$\frac{\partial H}{\partial u} = 0 \qquad (14b)$$

The conditions given by equation (5) still apply.

Solving eqs. (14a) and (14b) yields

$$\underline{\dot{\lambda}} = W_1 \underline{x} - A \underline{\lambda} \qquad (15a)$$

$$u^* = \frac{1}{w_3} \underline{b}^T \underline{\lambda} \qquad (15b)$$

Temporarily eliminating u^* produces the result

$$\left\{ \begin{array}{c} \underline{\dot{x}} \\ \underline{\dot{\lambda}} \end{array} \right\} = \hat{A} \left\{ \begin{array}{c} \underline{x} \\ \underline{\lambda} \end{array} \right\} + \left\{ \begin{array}{c} \underline{f} \\ \underline{0} \end{array} \right\} \qquad (16)$$

where

$$\hat{A} = \left[\begin{array}{c|c} A & \dfrac{1}{w_3} \underline{b}\,\underline{b}^T \\ \hline W_1 & -A^T \end{array} \right]$$

If equation (16) is now solved for $\underline{\lambda}$ in terms of $\underline{x}$ and of $\underline{f}$, equation (15b) will then furnish an expression for the optimal control u^*.

As noted before, optimal control texts generally treat the homogenous problem (where $\underline{f}(t) \equiv 0$), but they do not provide an analytical solution to the nonhomogeneous system described by equations (5) and (16). Salukvadze has treated the nonhomogeneous problem (refs. 4 and 5), but this difficult treatment seems largely to have remai~ ed either uncomprehended or under-appreciated. This method is especially well suited to low-frequency disturbance rejection, and has been applied below to the present problem.

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The homogeneous solution to equation (15), where $\underline{f} = \underline{0}$, is

$$\left\{ \begin{matrix} \underline{x} \\ \underline{\lambda} \end{matrix} \right\}_h = e^{\hat{A}t} \left\{ \begin{matrix} \underline{x}_0 \\ \underline{\lambda}_0 \end{matrix} \right\} \tag{17}$$

The four eigenvalues of $\hat{A}$ may be found to be, in ascending order of real parts,

$$\mu_1 = -\left[\frac{-\beta_1 + (\beta_1^2 - 4\beta_2)^{1/2}}{2} \right]^{1/2} \tag{18a}$$

$$\mu_2 = -\left[\frac{-\beta_1 - (\beta_1^2 - 4\beta_2)^{1/2}}{2} \right]^{1/2} \tag{18b}$$

$$\mu_3 = -\mu_1 \tag{18c}$$

$$\mu_4 = -\mu_2 \tag{18d}$$

where β_1 and β_2 are defined as follows:

$$\beta_1 = \frac{2k}{m} - \frac{c^2}{m^2} - \frac{\alpha w_{1b}}{mw_3} \tag{19a}$$

and

$$\beta_2 = \beta_1^2 - 4 \left[\frac{\alpha^2 w_{1a}}{m^2 w_3} + \frac{k}{m^2} \right] \tag{19b}$$

The eigenvectors of $\hat{A}$ corresponding to the respective eigenvalues μ_k may be chosen to be

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$$\underline{p}_k = \left\{ \begin{array}{c} 1 \\[1em] \mu_k \\[1em] \dfrac{\gamma_4}{\mu_k} + \dfrac{\gamma_1^2}{\gamma_3 \mu_k} + \dfrac{\gamma_1(\gamma_2 + \mu_k)}{\gamma_3)} \\[1.5em] \dfrac{\gamma_1 + (\gamma_2 + \mu_k)\mu_k}{\gamma_3} \end{array} \right\} \qquad (20a)$$

where γ_1, γ_2, γ_3, and γ_4, are defined below:

$$\gamma_1 = \frac{k}{m} \qquad (20b)$$

$$\gamma_2 = \frac{c}{m} \qquad (20c)$$

$$\gamma_3 = \frac{\alpha^2}{m^2 w_3} \qquad (20d)$$

$$\gamma_4 = w_{1a} \qquad (20e)$$

Using equations (18) through (20) with (17) the solution to the homogeneous system is

$$\left\{ \begin{array}{c} \underline{x} \\ \underline{\lambda} \end{array} \right\}_h = \left\{ \begin{array}{l} c_1 e^{\mu_1 t} \underline{p}_{1_1} + c_2 e^{\mu_2 t} \underline{p}_{2_1} + c_3 e^{-\mu_1 t} \underline{p}_{3_1} + c_4 e^{-\mu_2 t} \underline{p}_{4_1} \\[1em] c_1 e^{\mu_1 t} \underline{p}_{1_2} + c_2 e^{\mu_2 t} \underline{p}_{2_2} + c_3 e^{-\mu_1 t} \underline{p}_{3_2} + c_4 e^{-\mu_2 t} \underline{p}_{4_2} \end{array} \right\} \qquad (21)$$

with $\underline{p}_k = \left\{ \begin{array}{c} \underline{p}_{k_1} \\ \underline{p}_{k_2} \end{array} \right\}$, $k = 1, \ldots, 4$ and where $c_1, \ldots, c_4$ are arbitrary constants.

Application of the variation of parameters method with terminal conditions (eqs. (5b) and (c)) leads to the general solution of the nonhomogeneous system, with two constants of integration yet undermined.

If the two constants of integration are eliminated by solving for $\underline{\lambda}$ in terms of $\underline{x}$ and $\underline{f}$, the general solutions for λ_1 and λ_2 become:

$$\lambda_1 = \xi_1 x_1 + \xi_2 x_2 + \xi_3 e^{-\mu_1 t} + \xi_4 e^{-M_2 t} \tag{22a}$$

$$\lambda_2 = \xi_5 x_1 + \xi_6 x_2 + \xi_7 e^{-\mu_1 t} + \xi_8 e^{-\mu_2 t} \tag{22b}$$

in which the ξ_i's are functions of the eigenvalues and eigenvectors of $\hat{A}$, and of the disturbances $\underline{f}(t)$.

The Solution Form

Using the fact that

$$u^*(t) = \frac{1}{w_3} \underline{\lambda}^T \underline{b} \quad \left[\text{cf. eq. (15b)}\right] \tag{23}$$

the optimal control is found to be

$$u^*(t) = \eta_1 x_1 + \eta_2 x_2 + \eta_3 e^{-\mu_1 t} \int e^{\mu_1 t} f_2(t) dt + \eta_4 e^{-\mu_2 t} \int e^{\mu_2 t} f_2(t) dt \tag{24a}$$

where

$$\eta_1 = \frac{-m}{\alpha} \left(\frac{k}{m} - \mu_1 \mu_2 \right) \tag{24b}$$

$$\eta_2 = \frac{-m}{\alpha} \left(\frac{c}{m} + \mu_1 + \mu_2 \right) \tag{24c}$$

$$\eta_3 = \frac{m}{\alpha} \left(\frac{1}{\mu_1 - \mu_2} \right) \left(\mu_1^2 + \frac{c}{m} \mu_1 + \frac{k}{m} \right) \tag{24d}$$

$$\eta_4 = -\frac{m}{\alpha} \left(\frac{1}{\mu_1 - \mu_2} \right) \left(\mu_2^2 + \frac{c}{m} \mu_2 + \frac{k}{m} \right) \tag{24e}$$

(It should be noted that the feedback gains η_1 and η_2 are those which would result from applying standard LQR theory to the homogeneous system equation $\underline{\dot{x}} = A\underline{x} + b\underline{u}$). In

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equations (24a) to (f) μ_1, μ_2 are the eigenvalues of $\hat{A}$ with negative real parts, (see eqs. (18a) and (b))

$$f_2(t) = \frac{k}{m}d + \frac{c}{m}\dot{d}. \qquad (24f)$$

By repeated application of the method of integration by parts, the control may be re-expressed in terms of an infinite sum:

$$u^*(t) = \eta_1 x_1 + \eta_2 x_2 + \eta_3 \left[\sum_{r=0}^{\infty} \frac{(-1)^r f_2^{(r)}(t)}{\mu_1^{r+1}} \right] + \eta_4 \left[\sum_{r=0}^{\infty} \frac{(-1)^r f_2^{(r)}(t)}{\mu_2^{r+1}} \right] \qquad (25)$$

Rewriting f_2 in terms of d and $\dot{d}$, the control function becomes

$$u^*(t) = \eta_1 x(t) + \eta_2 \dot{x}(t) + \left[\frac{k}{m} \left(\frac{\eta_3}{\mu_1} + \frac{\eta_4}{\mu_2} \right) \right] d(t)$$

$$+ \sum_{i=1}^{r-1} \left[(-1)^{i-1} \frac{c}{m} \left(\frac{\eta_3}{\mu_1^i} + \frac{\eta_4}{\mu_2^i} \right) + (-1)^i \frac{k}{m} \left(\frac{\eta_3}{\mu_1^{i+1}} + \frac{\eta_4}{\mu_2^{i+1}} \right) \right] d^{(i)}(t) \qquad (26)$$

$$+ \left[(-1)^{n-1} \frac{c}{m} \left(\frac{\eta_3}{\mu_1^n} + \frac{\eta_4}{\mu_2^n} \right) \right] d^{(n)}(t) + \text{higher order terms}$$

This may be written in more appealing form as

$$u^*(t) = c_p x(t) + c_v \dot{x}(t) + c_{d0} d(t) + c_{d1} \dot{d}(t) + \text{higher order terms} \qquad (27)$$

in which the constant coefficients c_p, c_v, c_{d0}, and c_{d1} may be defined from equations (24) and (26). Clearly, if the infinite sums converge rapidly enough, the optimal control can be approximated by

$$u^*(t) = c_p x(t) + c_v \dot{x}(t) + c_{d0} d(t) + c_{d1} \dot{d}(t) \qquad (28)$$

For very low frequency disturbances the higher order terms in equation (26) are negligibly small, and the control (eq. (28)) closely approximates the optimal. If, in fact, the second- and

higher-order derivatives of $d(t)$ are identically zero, the approximation is exact. It can be shown that for the critically damped closed loop system the eigenvalues are real and equal, and the convergence is more rapid than for the overdamped system. Further, as the closed-loop system eigenvalues become more negative the convergence speed goes up as well.

5. CONTROL EVALUATION

Physical Realizability of the Control

The control, equation (25), is physically realizable, if the states and sufficient derivatives of $d(t)$ are accessible (or estimable by an observer), and if the higher order terms are negligible. It is not necessary that the eigenvalues be real, although the proof of this requires a more general linear algebra or state-transition-matrix approach.

If values are assigned to the system parameters, associated controller gains can be evaluated. Suppose that $m = 100$ lbm, $k = 0.3$ lbf/ft, $c = 0$ lbf-sec/ft, and $\alpha = 10$ lbf/Amp. With w_3 arbitrarily set at 1 and w_{1b} varied, associated integer values of w_{1a} can be found below which the eigenvalues μ_1 and μ_2 will always be real. Such values are tabulated in table I. Stated otherwise, the tabulated values of the weights w_{1a} and w_{1b} are those integer values (for the sake of simplicity) for which the closed loop system is closest to being critically damped without being undamped. Corresponding controller feedback and feedforward gains (for the first five derivatives) are also included.

The states $x(t)$ and $\dot{x}(t)$ and the derivatives $d^{(0)}(t)$, $d^{(1)}(t)$, and $d^{(2)}(t)$ are clearly available for an Earth-based system. However, in space, the only absolute measurements which can be directly available are $\ddot{x}(t)$ and $\ddot{d}(t)$, from which $\dot{x}(t)$, $\dot{d}(t)$ and $x(t)$, $d(t)$ are obtainable only by successive integration(s). Rearrangements of (28) into

$$u^*(t) = (c_p + c_{d0})x(t) + (c_v + c_{d1})\dot{x}(t) - c_{d0}[x(t) - d(t)] - c_{d1}[\dot{x}(t) - \dot{d}(t)] \qquad (29)$$

or

$$u^*(t) = (c_p + c_{d0})d(t) + (c_v + c_{d1})\dot{d}(t) + c_p[x(t) - d(t)] + c_v[\dot{x}(t) - \dot{d}(t)] \qquad (30)$$

obviates the need for one accelerometer, but one accelerometer plus two integrations remain necessary for either the platform or the experiment. Since $[x(t) - d(t)]$ (or one of its integrals) has not been weighted in the performance index J, experiment drift will be a problem that must be corrected either by another control loop or by a change of system states. The latter could be accomplished by incorporating an accelerometer attached to the experiment into the state equation. Alternatively, one could append an integrator to the plant, include the current $i(t)$ as a third state, and optimize the control di/dt. But for the sake of simplicity (i.e., fewer states) the former has been assumed (without development) in this paper.

The higher order terms of the control (eqs. (25) and (26)) can be neglected, for low frequencies, if the eigenvalues μ_1 and μ_2 are of sufficient modulus. These eigenvalues, in turn,

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are under the control of the designer, determined by chosen weights w_{1a}, w_{1b}, and w_3. It is apparent from equation (25) that $u'(t)$ essentially reduces to two alternating power series. For a sinusoidal disturbance of frequency ω the series form of the control converges for $|\omega/\mu_i| < 1 (i = 1,2)$.

It can be shown that each alternating power series converges like $\displaystyle\sum_{r=0}^{\infty} (-1)^r \left(\frac{\omega}{\mu}\right)^{2r}$. With "low" frequency disturbances (i.e., small relative to system closed loop eigenvalues) a control formed by series truncation very closely approximates the optimal.

For example, suppose that the normalized frequencies $|\omega/\mu_i|$ for sinusoidal disturbances are less than 1/5, and that only the feedforward control terms $c_{d0}d(t)$ and $c_{d1}d(t)$ are included with the feedback terms. Even so, the feedforward portion of the truncated control, at any time t, will be a current that is still within 4 percent (i.e., $(1/5)^2$) of the feedforward portion of the actual optimal. If the normalized frequencies are below 1/10, this approximation error will be less than 1 percent. Table 1 shows that the gains c_{di} of higher order derivatives $d^{(i)}(t)$ (see equation (26) for the algebraic representations) are, in fact, quite small.

In some circumstances there may be design constraints which prevent the designer from selecting weights that lead to sufficiently rapid convergence. However, since convergence occurs rapidly even for eigenvalues of relatively small modulus ($|\omega/\mu_i| < 1/3$), in a great many cases the designer will have much latitude in choice of weights. For "low" frequency disturbances, in these cases, a control which includes only one or two feedforward terms will be "close" to the optimal. These frequencies be well attenuated.

Higher frequency disturbances will also be well attenuated, <u>provided</u> the input-to-output transfer functions(s) are at least strictly proper in the Laplace Transform variables. This will <u>not</u> be the case for the present problem if more than three feedforward gains (c_{d0}, c_{d1}, c_{d2}) are included in the control. Practically, this means that only proportional and first-derivative feedforward (equation (25) with r = 0,1 or equation (26) with n = 2) should be added to the feedback control terms. As will be seen shortly, however, adding even the proportional feedforward terms(s) can dramatically improve the disturbance rejection over that afforded by LQR feedback alone.

Transfer Function and Block Diagram

Neglecting the higher order terms, the transfer function between input and output accelerations or displacements is

$$\frac{s^2 X(s)}{s^2 D(s)} = \frac{X(s)}{D(s)} = \frac{\left(\dfrac{c}{\alpha} - c_{d1}\right)s + \left(\dfrac{k}{\alpha} - c_{d0}\right)}{\left(\dfrac{m}{\alpha}\right)s^2 + \left(\dfrac{c}{\alpha} + c_v\right)s + \left(\dfrac{k}{\alpha} + c_p\right)}, \tag{31}$$

and a block diagram of the controlled system can be drawn as in figure 2.

Control Stability, Stability Robustness, and General Robustness

Since the control feedback gains are the same as those obtained by solution of the standard Linear Quadratic Regulator (LQR) problem, the closed loop system is stable and enjoys the stability robustness characteristics guaranteed by the (LQR) approach to optimality, viz., a minimum of 60° phase margin, infinite positive gain margin, and 6 dB negative gain margin (ref. 6). Additionally, numerical checks indicate that it enjoys substantial insensitivity, or general robustness to uncertainties in k, c, and m, as indicated by table II and figures 3 to 10. By comparing the Bode plots of figures 3, 5, 7, and 9 (corresponding to controls using both LQR feedback (F/B) and proportional feedforward (F/F) with those of figures 4, 6, 8, and 10, respectively (corresponding to controls using LQR F/B only), one can see that adding feedforward substantially improves disturbance rejection at low frequencies. For example a comparison of figures 3 with figure 4 indicates that the optimal control method described above can lead to acceleration reductions of greater than four orders of magnitude for all frequencies. This reduction is more than two orders of magnitude below that afforded by LQR feedback alone at the lower frequencies, i.e., those most heavily weighted in the performance index.

The order of the reduction is eventually limited by control cost, of course, probably in terms either of actuator-related limitations (such as heat removal or force generation requirements) or of power limitations (especially in a space station environment). The control also leads to displacement reductions of the same magnitude, limited in this case by actuator-stroke or spatial limitations. Providing a unit of transmissibility for very low frequencies and/or weighting $\int(x - d)$ in the performance index J, would be steps toward addressing these latter limitations.

Computational Aspects

A significant amount of algebra was required to solve the two-state problem of this paper, and the labor involved increases dramatically with each additional state. However, such symbolic manipulators as MACSYMA may be used to ease the workload if a symbolic solution is desired. Further, well-known numerical methods exist (i.e., Potter's method (ref. 7) or Laub's method (ref. 8)) for solving the homogeneous system. These can readily provide the feedback gains in numerical form, even for problems with many states. It might be anticipated, then, that a numerical method also exists for finding the desired feedforward gains.
Such is the case, as will be shown in a later paper.

6. CONCLUSIONS

This paper has applied an existing method for obtaining an optimal control to the microgravity platform isolation problem, for which the disturbances to be rejected are low-frequency accelerations. The system was assumed to be representable in the form $\dot{\underline{x}} = A\underline{x} + \underline{b}u + \underline{f}$,

with quadratic cost function $J = \dfrac{1}{2} \int_0^\infty (\underline{x}^T W_1 \underline{x} + w_3 u^2)dt$ and diagonal weighing matrix W_1.

The resultant control law was found to be simple, stable, robust, and physically realizable. Further it was shown to have excellent acceleration- and displacement-attenuation characteristics, and to be frequency-weighted toward the low end of the acceleration spectrum.

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The method is extendable to the case for which only relative positions and velocities, and absolute accelerations, are available; and can be applied so as to weight relative displacements in the performance index.

The approach as presented is algebraically intensive, but symbolic manipulators can be used to ease the algebraic labors. Further, since the method produces feedback gains identical to those obtained by the LQR approach to optimality, numerical computation of those gains is easily accomplished, even for large systems. The feedforward gains can be found numerically with comparable ease.

7. ACKNOWLEDGEMENTS

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TABLE I. - OPTIMAL F/F AND F/B GAINS FOR SELECTED STATE VARIABLE AND CONTROL WEIGHTINGS

[System parameters m = 100 lbm. k = 0.3 lbf/ft, c = 0.000622 lbf-sec/ft (ζ = 0.1 percent), α = 10 lb/A.]

Weights			F/B Gains		F/F Gains					
w_{1a}	w_{1b}	w_3	C_p	C_v	C_{d0}	C_{d1}	C_{d2}	C_{d3}	C_{d4}	C_{d5}
2	1	1	1.3845	1.3637	0.0294	-0.0006	-0.0070	-0.0067	-0.0049	-0.0032
10	2	1	3.1324	1.9863	.0297	-.0001	-.0030	-.0019	-.0009	-.0004
23	3	1	4.7659	2.4413	.0298	-.0000	-.0020	-.0010	-.0004	-.0001
41	4	1	6.3732	2.8210	.0299	.0000	-.0015	-.0007	-.0002	-.0001
64	5	1	7.9701	3.1544	.0299	.0000	-.0012	-.0005	-.0001	-.0000
92	6	1	9.5617	3.4552	.0299	.0000	-.0010	-.0004	-.0001	-.0000
126	7	1	11.1950	3.7354	.0299	.0000	-.0008	-.0003	-.0001	-.0000
165	8	1	12.8153	3.9949	.0299	.0000	-.0007	-.0002	-.0001	-.0000
209	9	1	14.4269	4.2380	.0299	.0000	-.0006	-.0002	-.0000	-.0000
258	10	1	16.0324	4.4674	.0299	.0000	-.0006	-.0002	-.0000	-.0000
581	15	1	24.0740	5.4729	.0300	.0001	-.0004	-.0001	-.0000	-.0000
1034	20	1	32.1259	6.3209	.0300	.0001	-.0003	-.0001	-.0000	-.0000
1617	25	1	40.1819	7.0680	.0300	.0001	-.0002	-.0000	-.0000	-.0000
2329	30	1	48.2297	7.7431	.0300	.0001	-.0002	-.0000	-.0000	-.0000
3171	35	1	56.2816	8.3640	.0300	.0001	-.0002	-.0000	-.0000	-.0000
4143	40	1	64.3361	8.9420	.0300	.0001	-.0001	-.0000	-.0000	-.0000
9325	60	1	96.5360	10.9526	.0300	.0001	-.0001	-.0000	-.0000	-.0000
16581	80	1	128.7372	12.6475	.0300	.0001	-.0001	-.0000	-.0000	-.0000
25911	100	1	160.9389	14.1407	.0300	.0001	-.0001	-.0000	-.0000	-.0000

TABLE II. - CLOSED LOOP TRANSFER FUNCTIONS FOR SYSTEM WITH DESIGN PARAMETER VALUES

[k = 0.3, c = 0.00622, and m = 100; G1, G3, G5, and G7 include both LQR F/B and proportional F/F; G2, G4, G6, and G8 include LQR F/B alone. Weighting parameters used were, $w_{1a} = 258$, $w_{1b} = 10$, $w_3 = 1$ (table I).]

System parameter			Closed loop transfer function $\dfrac{s^2X(s)}{s^2D(s)}$
$k\left(\dfrac{lbf}{ft}\right)$	$c\left(\dfrac{lbf-sec}{ft}\right)$	m(lbm)	
0.3	0.000622 ($\zeta = 0.1\%$)	100	$G1(s) = \dfrac{0.0000622s + 0.0001}{0.31056s^2 + 4.4675s + 16.0624}$
0.3	0.000622	100	$G2(s) = \dfrac{0.0000622s + 0.0300}{0.31056s^2 + 4.4675s + 16.0624}$
0.45	0.000622	100	$G3(s) = \dfrac{0.0000622s + 0.0151}{0.31056s^2 + 4.4675s + 16.0774}$
0.45	0.000622	100	$G4(s) = \dfrac{0.0000622s + 0.0450}{0.31056s^2 + 4.4675s + 16.0774}$
0.3	0.00622	100	$G5(s) = \dfrac{0.000622s + 0.0001}{0.31056s^2 + 4.4680s + 16.0624}$
0.3	0.00622	100	$G6(s) = \dfrac{0.000622s + 0.0300}{0.31056s^2 + 4.4680s + 16.0624}$
0.45	0.00622	90	$G7(s) = \dfrac{0.000622s + 0.0151}{0.27950s^2 + 4.4680s + 16.0774}$
0.45	0.00622	90	$G8(s) = \dfrac{0.000622s + 0.0450}{0.27950s^2 + 4.4680s + 16.0774}$

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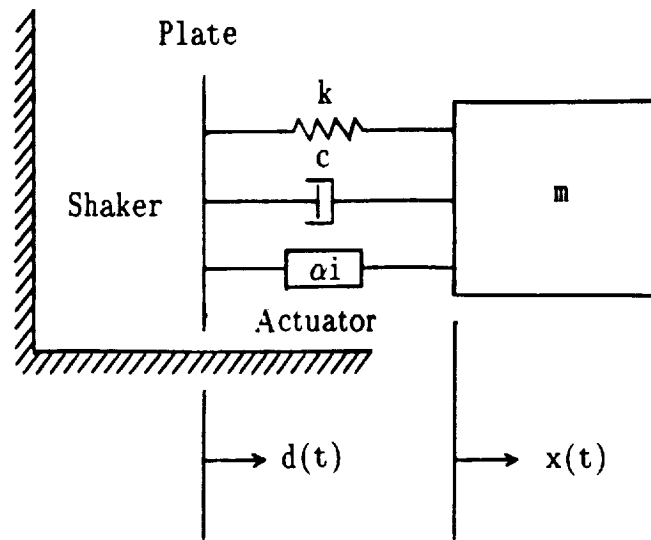
FIGURE 1. - SYSTEM MODEL.
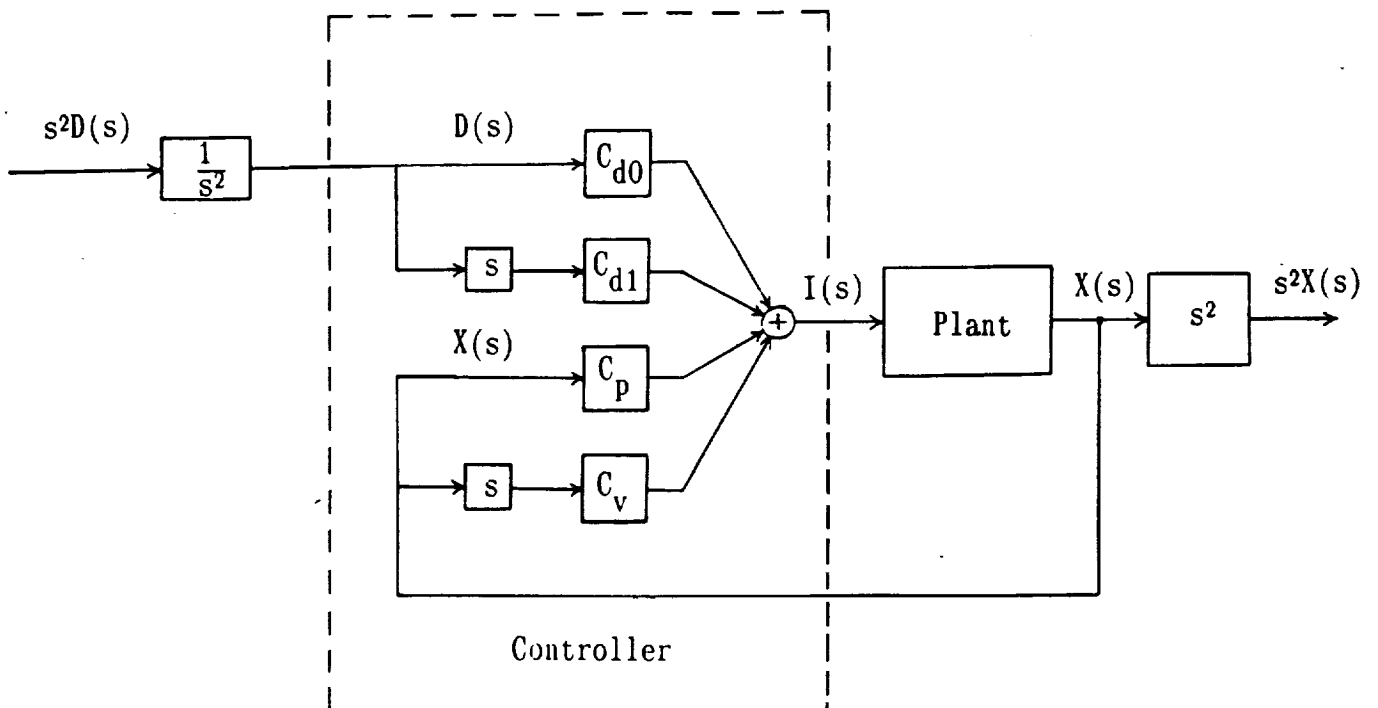
FIGURE 2. - BLOCK DIAGRAM.

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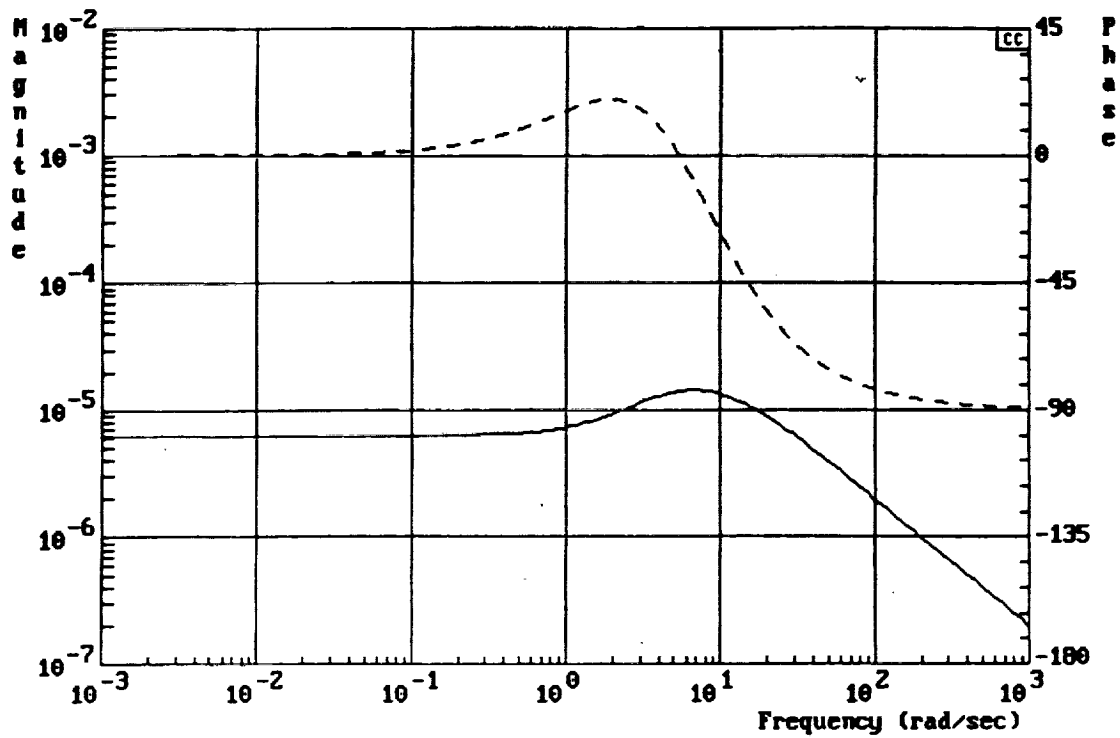
FIGURE 3. - OPTIMAL CONTROL WITH FEEDBACK AND FEEDFORWARD. SYSTEM KNOWN EXACTLY.
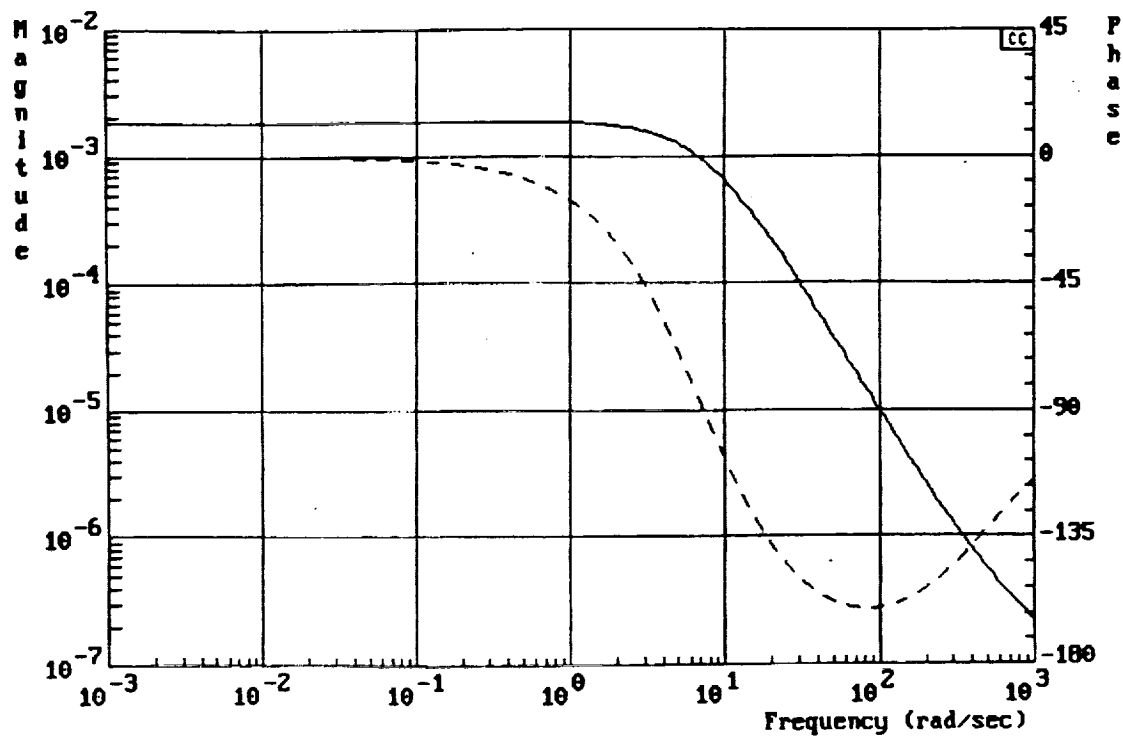
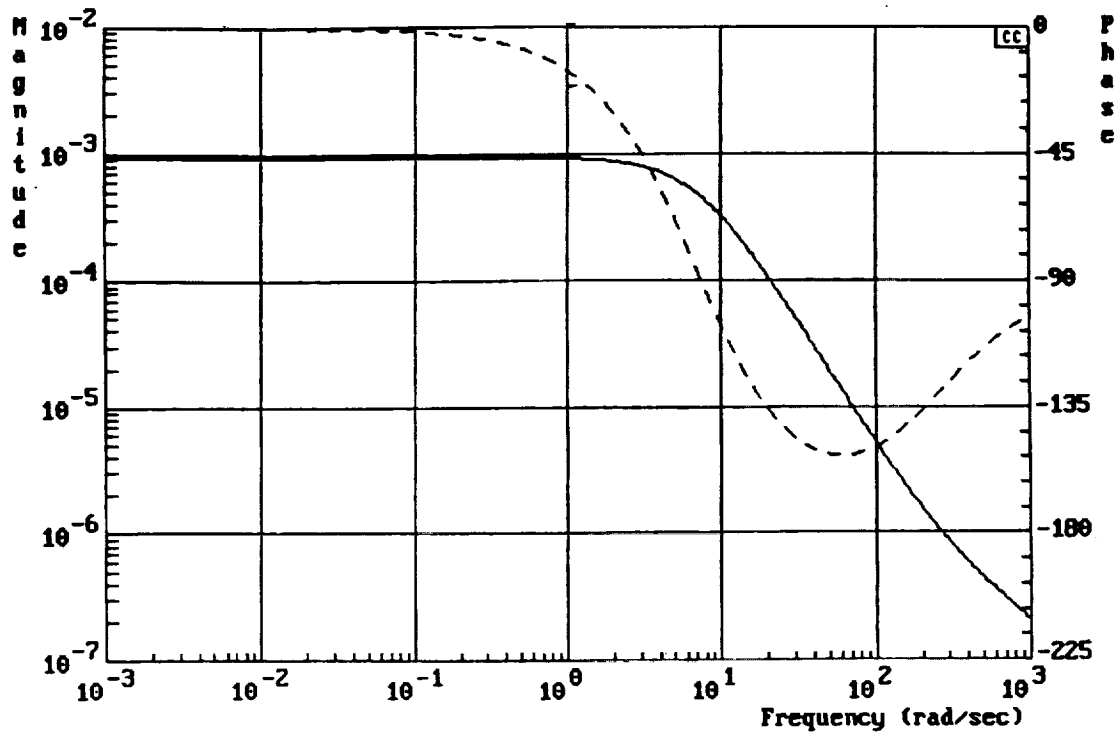FIGURE 4. - OPTIMAL CONTROL WITH FEEDBACK ONLY. SYSTEM KNOWN EXACTLY.

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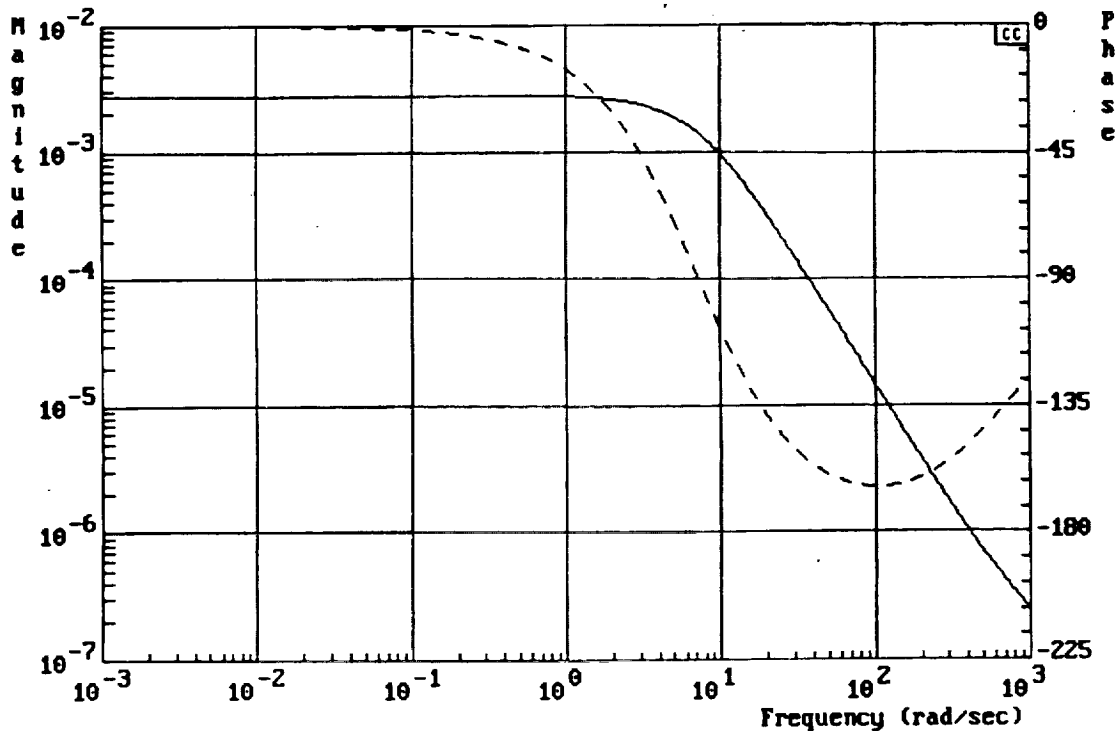FIGURE 5. - OPTIMAL CONTROL WITH FEEDBACK AND FEEDFORWARD, SYSTEM STIFFNESS ESTIMATE POOR.

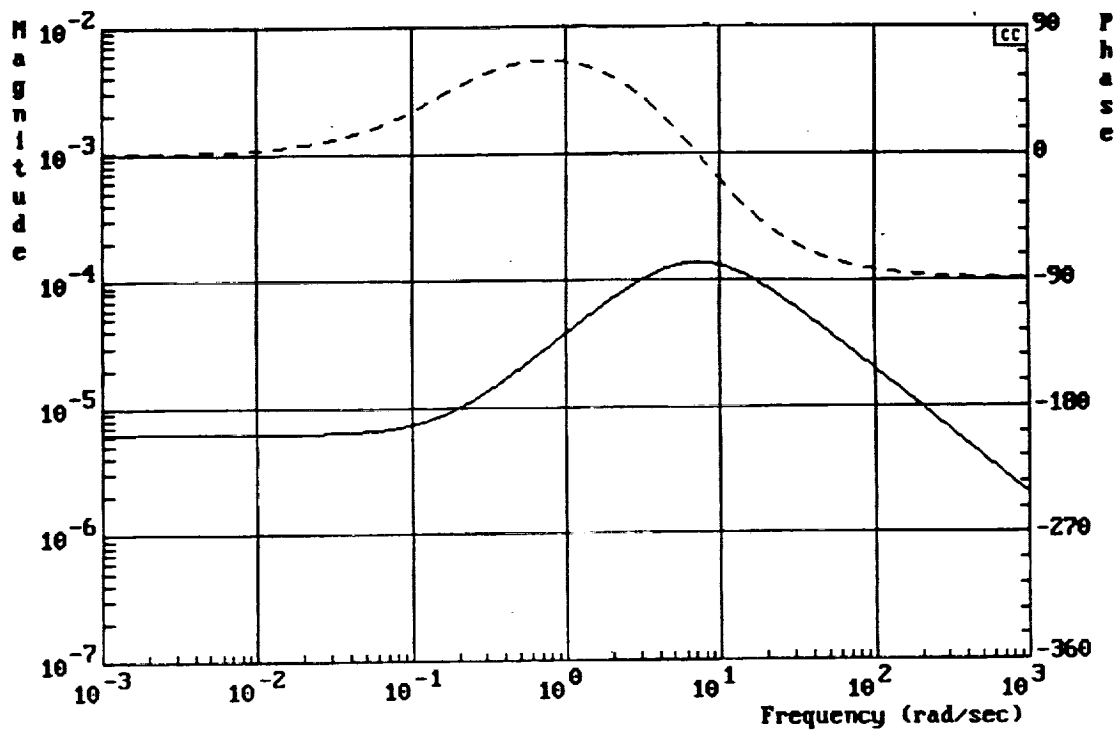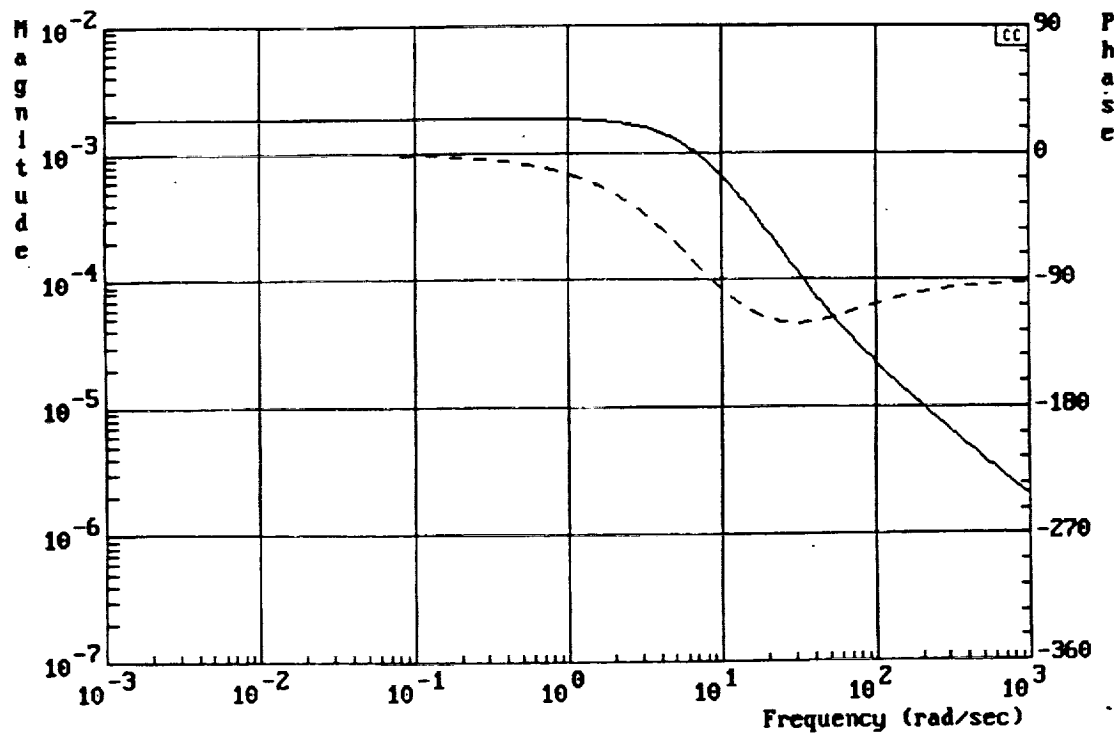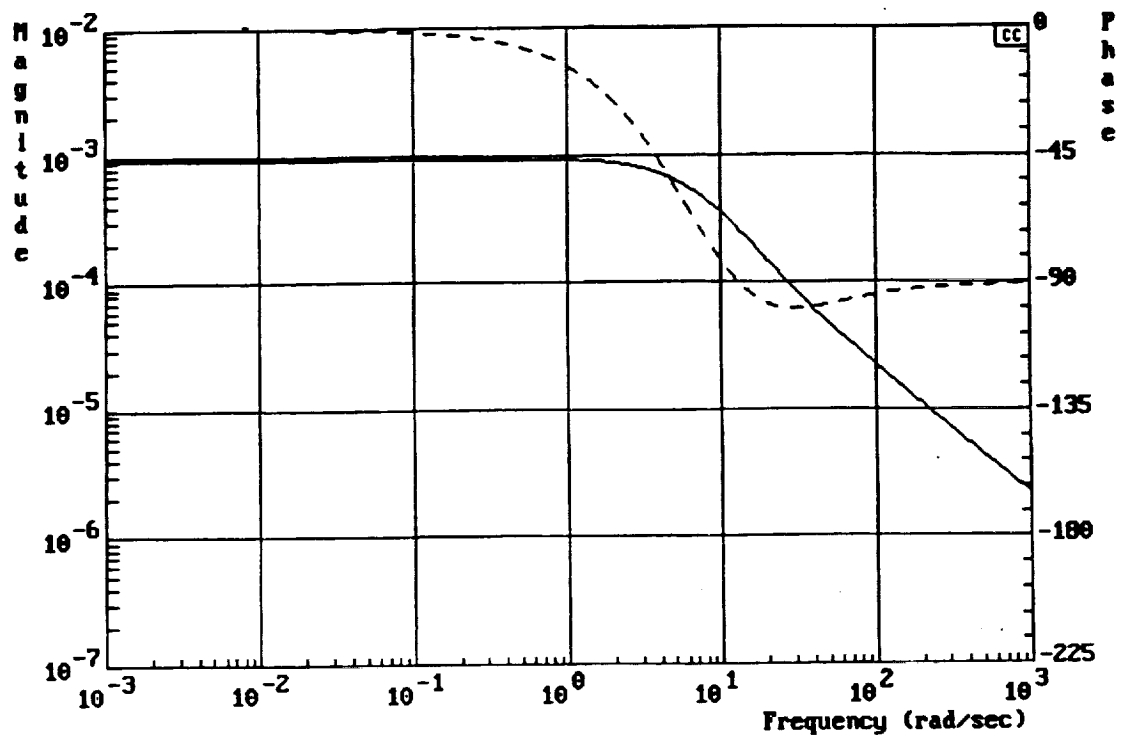FIGURE 6. - OPTIMAL CONTROL WITH FEEDBACK ONLY, SYSTEM STIFFNESS ESTIMATE POOR.

FIGURE 7. - OPTIMAL CONTROL WITH FEEDBACK AND FEEDFORWARD, SYSTEM DAMPING ESTIMATE POOR.
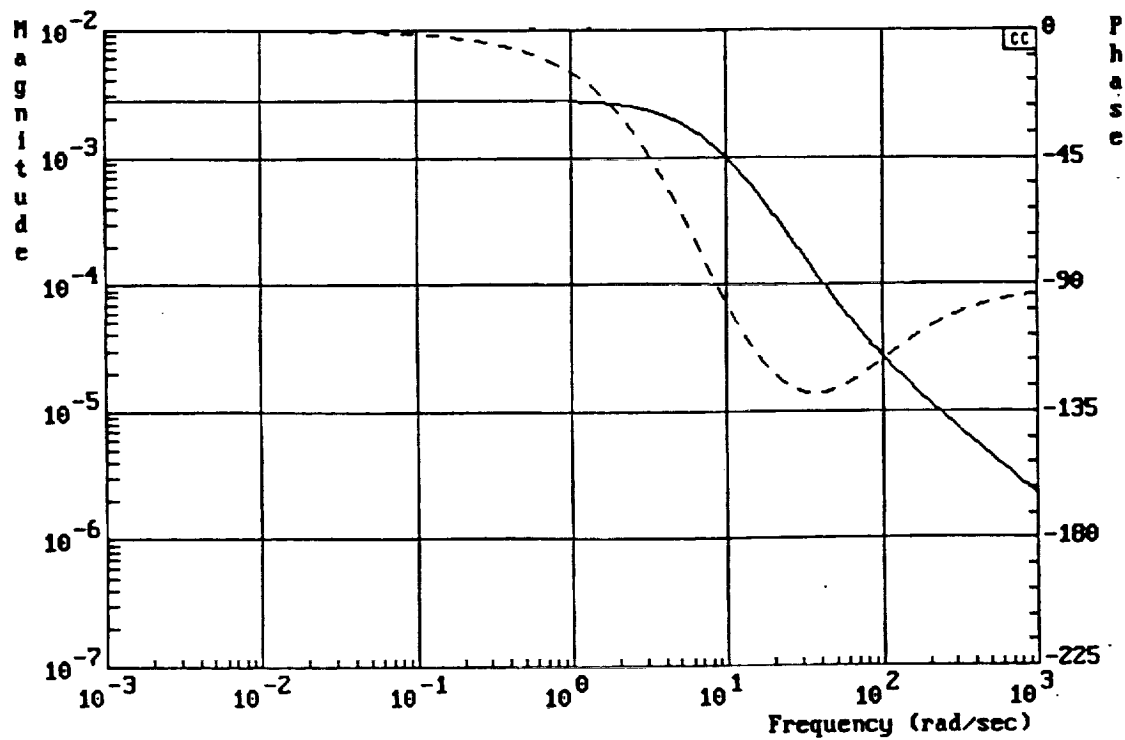

FIGURE 8. - OPTIMAL CONTROL WITH FEEDBACK ONLY, SYSTEM DAMPING ESTIMATE POOR.

FIGURE 9. - OPTIMAL CONTROL WITH FEEDBACK AND FEEDFORWARD, SYSTEM k, c, m ESTIMATES POOR.

FIGURE 10. - OPTIMAL CONTROL WITH FEEDBACK ONLY, SYSTEM k, c, m ESTIMATES POOR.

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13. ABSTRACT (Maximum 200 words)

Certain experiments contemplated for space platforms must be isolated from the accelerations of the platform. In this paper an optimal active control is developed for microgravity vibration isolation, using constant state feedback gains (identical to those obtained from the Linear Quadratic Regulator [LQR] approach) along with constant feedforward (preview) gains. The quadratic cost function for this control algorithm effectively weights external accelerations of the platform disturbances by a factor proportional to $(1/\omega)^4$. Low frequency accelerations (less than 50 Hz) are attenuated by greater than two orders of magnitude. The control relies on the absolute position and velocity feedback of the experiment and the absolute position and velocity feedforward of the platform, and generally derives the stability robustness characteristics guaranteed by the LQR approach to optimality. The method as derived is extendable to the case in which only the relative positions and velocities and the absolute accelerations of the experiment and space platform are available.

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